GMAT® Quant

Concepts & Formulae

1) Number Properties

i) Integers

Numbers, such as -1, 0, 1, 2, and 3, that have no fractional part. Integers include the counting numbers (1, 2, 3, …), their negative counterparts (-1, -2, -3, …), and 0.

ii) Whole & Natural Numbers

The terms from 0,1,2,3,….. are known as Whole numbers. Natural numbers do not include 0.

iii) Factors

Positive integers that divide evenly into an integer. Factors are equal to or smaller than the integer in question. 12 is a factor of 12, as are 1, 2, 3, 4, and 6.

iv) Factor Foundation Rule

If *a* is a factor of *b*, and *b* is a factor of *c*, then *a* is also a factor of *c*. For example, 3 is a factor of 9 and 9 is a factor of 81. Therefore, 3 is also a factor of 81.

v) Multiples

Multiples are integers formed by multiplying some integer by any other integer. For example, 6 is a multiple of 3 (2 * 3), as are 12 (4 * 3), 18 (6 * 3), etc. In addition 3 is also a multiple of itself i.e. 3 (1*3). Think of multiples as equal to or larger than the integer in question

vi) Prime Numbers

A positive integer with exactly two factors: 1 and itself. The number 1 does not qualify as prime because it has only one factor, not two. The number 2 is the smallest prime number; it is also the only even prime number. The numbers 2, 3, 5, 7, 11, 13 etc. are prime.

vii) Prime Factorization

Prime factorization is a way to express any number as a product of prime numbers. For example, the prime factorization of 30 is 2 * 3 * 5. Prime factorization is useful in answering questions about divisibility.

viii) Greatest Common Factor

Greatest Common FACTOR refers to the largest factor of two (or more) integers. Factors will be equal to or smaller than the starting integers. The GCF of 12 and 30 is 6 because 6 is the largest number that goes into both 12 and 30.

viii) Least Common Multiple (LCM)

Least Common Multiple refers to the smallest multiple of two (or more) integers. Multiples will be equal to or larger than the starting integers. The LCM of 6 and 15 is 30 because 30 is the smallest number that both 6 and 15 go into.

ix) Odd & Even Numbers

Any number divisible by 2 is even and not divisible by 2 is odd.

Odd & Even number Rules

Function	**Result**
even + even	even
even + odd	odd
odd + odd	even
even - even	even
even - odd	odd
odd - odd	even
even * even	even
even * odd	even
odd * odd	odd
even ÷ even	anything (even, odd, or not an integer)
even ÷ odd	even or not an integer
odd ÷ even	not an integer
odd ÷ odd	odd or not an integer

Note:

Division rules are more complicated because an integer answer is not always guaranteed. If the result of the division is not an integer, then that result cannot be classified as either even or odd.

x) Absolute Value

The distance from zero on the number line. A positive number is already in the same form as that number's absolute value. Remove the negative sign from a negative number in order to get that number's absolute value. For example the absolute value of $|-2|$ is 2.

xi) Positive-Negative Number Rules

Function	**Result**
positive * positive	positive
positive * negative	negative
negative * negative	positive
positive ÷ positive	positive
positive ÷ negative	negative
negative ÷ negative	positive

xii) Product of n consecutive integers and divisibility

The product of n consecutive integers is always divisible by n! Given 5*6*7*8, we have n = 4 consecutive integers. The product of 5*6*7*8 (=1680), therefore, is divisible by 4! = 4*3*2*1 = 24.

xiii) Sum of n consecutive integers and divisibility

There are two cases, depending upon whether n is odd or even:

- If n is odd, the sum of the integers is <u>always</u> divisible by n. Given 5+6+7, we have n = 3 consecutive integers. The sum of 5+6+7 (=18), therefore, is divisible by 3.

- If n is even, the sum of the integers is <u>never</u> divisible by n. Given 5+6+7+8, we have n = 4 consecutive integers. The sum of 5+6+7+8 (=26), therefore, is <u>not</u> divisible by 4.

xiv) PEMDAS

First, perform all operations that are inside parentheses. Absolute value signs also fall into this category. In addition, for any expression with fractions, add parentheses around each distinct fraction.

Second, simplify any exponents that appear in the expression.

Third, perform any multiplication and division in the expression; if there are more than one of these, perform the operations from left to right in the expression.

Fourth, perform any addition and subtraction in the expression; if there are more than one of these, perform the operations from left to right in the expression.

2) Base & Exponent

In the expression b^n, the variable b represents the base and n represents the exponent. The base is the number that we multiply by itself n times. The exponent indicates how many times to multiple the base, b, by itself. For example, $2^3 = 2 * 2 * 2$, or 2 multiplied by itself three times.

Equations that include an exponent are called as exponential equations. When solving equations with even exponents, we must consider both positive and negative possibilities for the solutions. For example, for $x^2 = 25$, the two possible solutions are 5 and -5.

i) Base of Zero

An exponential expression with base 0 yields 0, regardless of the exponent. $0^{12} = 0$.

ii) Base of One

An exponential expression with base 1 yields 1, regardless of the exponent. $1^{12} = 1$.

iii) Base of Negative One

An exponential expression with base -1 yields 1 when the exponent is even and -1 when the exponent is odd. $(-1)^{15} = -1$ and $(-1)^{16} = 1$.

iv) Fractional Base

When the base is a fraction between zero and one, the value decreases as the exponent increases. $(1/3)^3 = 1/3 * 1/3 * 1/3 = 1/27$, which is smaller than the starting fraction, 1/3.

v) Compound Base

When the base represents a product (multiplication) or quotient (division), we can choose to multiply or divide the base first and then raise the base to the exponent, or we can distribute the exponent to each number in the base. For example $(3 * 4)^2 = 12^2 = 144$ *OR* $(3*4)^2 = 3^2 * 4^2 = 9 * 16 = 144$.

vi) Exponent of Zero

Any non-zero base raised to the 0 yields 1. Eg. $15^0 = 1$.

vii) Exponent of One

Any based raised to the exponent of 1 yields the original base. Eg. $15^1 = 15$.

viii) Negative Exponents

Put the term containing the exponent in the denominator of a fraction and make the exponent positive. For example $4^{-2} = (1/4)^2$

ix) Fractional Exponents

If the exponent is a fraction, the numerator reflects what power to raise the base to, and the denominator reflects which root to take. For example $4^{2/3}$ = CUBE ROOT (4^2).

x) Simplification Rules for Exponents

Rule	**Result**
$3^4 * 3^3$ = Add the exponents	3^7
$3^4 / 3^2$ = Subtract the exponents	3^2
$(3^4)^3$ = Multiply the exponents	3^{12}

xi) Root/Radical

The opposite of an exponent (in a sense). For example, $\sqrt{25}$ means what number (or numbers), when multiplied by itself twice, will yield 25?

Perfect square roots will yield an integer. Eg. $\sqrt{25} = 5$. Imperfect square roots do not yield an integer. $\sqrt{30}$ is not an integer, but it is between $\sqrt{25}$ and $\sqrt{36}$, or between 5 and 6.

xii) Simplifying Roots

Roots can be combined or split apart if the operation between the terms is multiplication or division. $\sqrt{(4 * 9)} = \sqrt{4} * \sqrt{9}$.

Note: If the operation between the terms is addition or subtraction, you cannot separate or combine the roots! $\sqrt{(4 + 9)}$ DOES NOT EQUAL $\sqrt{4} + \sqrt{9}$.

3) Equations & Inequalities

i) Equation

A combination of mathematical expressions and symbols that contains an 'equals' sign. Eg. 2 + 5 = 7 is an equation, as is $x + y = 5$

ii) Linear Equation

An equation that does not contain exponents or multiple variables multiplied together. $x + y = 5$ is a linear equation whereas $x*y = 5$ and $y = x^2$ are not. When plotted on a coordinate plane, linear equations will give you straight lines.

iii) Simultaneous Equation

These are two or more distinct equations containing two or more variables.

iv) Quadratic Equation

An expression including a variable raised to the second power (and no higher powers). Commonly of the form $ax^2 + bx + c$, where a, b, and c are constants.

v) Special Simplification cases

- $a^2 – b^2 = (a + b) * (a - b)$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 – 2ab + b^2$

vi) Sequence

A sequence is a collection of numbers in a set order. {3, 5, 7, 9, 11, …} is an example of a sequence for which the first five terms are specified (but the sequence continues beyond these five terms, as indicated by the "…")

vii) Linear/Arithmetic Sequence

A sequence in which the difference between successive terms is always the same. A constant number (which could be negative!) is added each time. Also called Arithmetic Sequence. Eg. {1,3,5,7,....}

viii) Exponential Sequence

A sequence in which the ratio between successive terms is always the same; a constant number (which could be positive or negative) is multiplied each time. Also called Geometric sequence. Eg. {2,4,8,16,.....}

ix) Functions

A rule or formula which takes an input (or given starting value) and produces an output (or resulting value). For example, $f(x) = x + 5$ represents a function, where x is the input, $f(x)$ is read as "f as a function of x" or "f of x" and refers to the output (also known as the "y" value), and $x + 5$ is the rule for what to do to the x input. Eg. $f(4) = x + 5 = 4 + 5 = 9$.

x) Domain

All of the possible inputs, or numbers that can be used for the independent variable, for a given function. In the function $f(x) = x^3$, the domain is all numbers.

xi) Range

All of the possible outputs, or numbers that can be used for the dependent variable, for a given function. In the function $f(x) = x^3$, the range is $f(x) >= 0$.

xii) Compound/Composite Functions

Two nested functions which are to be solved starting from the inner parentheses. For example, $f(g(x))$ is an example of a compound function and is read as "f of g of x." Given $f(x) = x + 5$ and $g(x) = 3x$, $g(x)$ is substituted first followed by f(x). Eg. $f(g(2))$, g(2) = 3*2 = 6 and now f(6) = 6+5 = 11.

xiii) Direct Proportion

Two given quantities are said to be "directly proportional" if the two quantities always change by the same factor and in the same direction. For example, doubling the input causes the output to double as well. The standard formula is $y = kx$, where x is the input, y is the output, and k is the proportionality constant (or the factor by which the numbers change).

xiv) Inverse Proportion

Two given quantities are said to be "indirectly proportional" if the two quantities change by reciprocal factors. For example, doubling the input causes the output to halve. Tripling the input cuts the output to one-third of its original value. The standard formula is $y = (k/x)$, where x is the input, y is the output and k is the proportionality constant

xv) Inequality

A comparison of quantities that have different values. There are four ways to express inequalities: less than (<), less than or equal to (<=), greater than (>), or greater than or equal to (=>).

Inequalities can be manipulated in the same way as equations with one exception: when multiplying or dividing by a negative number, the inequality sign flips.

xvi) Compound Inequality

This involves more than two inequalities strung together. For example, $2 < x < 5$ is a compound inequality, as is $a < b < c < d$. When manipulating, the same operations must be done to every term in the inequality. For example, given $x < y + 2 < 2x$, if we subtract 2 from the middle term to get y by itself, we must also subtract 2 from the first and third terms: $x - 2 < y < 2x – 2$.

4) Geometry

i) Polygon

A two-dimensional, closed shape made of line segments. For example, a triangle is a polygon, as is a rectangle. A circle is a closed shape but it is not a polygon because it does not contain line segments.

ii) Interior Angles

The angles that appear in the interior of a closed shape. The sum of those angles depends only upon the number of sides ‘n’ in the closed shape: (n - 2) * 180 = sum of interior angles, where n = the number of sides in the shape. For example, the interior angles of a four-sided closed shape will always add up to (4 - 2)*180 = 360°

iii) Perimeter & Area

In a polygon, the sum of the lengths of the sides is called the perimeter. The formula depends on the specific shape.

iv) Triangle

A three-sided closed shape composed of straight lines; the interior angles add up to 180°.

v) Vertex/Vertices

An "angle" or place where two lines of a shape meet; for example, a triangle has three vertices and a rectangle has four vertices.

vi) Right Triangle

A triangle that includes a 90°, or right, angle.

vii) Hypotenuse

The longest side of a right triangle. The hypotenuse is opposite to the right angle.

viii) Area of a Triangle

Area of Triangle = (base * height) / 2, where the base refers to any side of the triangle and the height refers to the length of a line drawn from the opposite vertex to create a 90° angle with that base.

ix) Pythagoras Theorem

A formula used to calculate the sides of a right triangle. According to this theorem $a^2 + b^2 = c^2$, where a and b are the lengths of the two legs of the triangle and c is the length of the hypotenuse of the triangle.

x) Isosceles Triangle

A triangle in which two of the three angles are equal; in addition, the sides opposite the two angles are equal in length.

xi) Equilateral Triangle

A triangle in which all three angles are equal and all three sides are of equal length.

xii) Area of an Equilateral Triangle

In addition to the standard area formula for triangles, equilateral triangles have a special formula for Area = $S^2 * \sqrt{3} / 4$, where S is the length of any side of the equilateral triangle.

xiii) Relationship between the Sides of a Triangle

The length of any side of a triangle must be larger than the positive difference between the other two sides, but smaller than the sum of the other two sides.

For example, given a triangle with sides 3, 4, and 5, 4 is an acceptable length because it is larger than the positive difference of 5 and 3 (5 - 3 = 2), and it is also smaller than the sum of 5 and 3 (5 + 3 = 8). By contrast, given a triangle with sides 3, 4, and 8, 8 is not an acceptable length because it is larger than the sum of the other two sides (3 + 4 = 7). The dimensions 3, 4, and 8, then, do not form a triangle.

xiv) Similar Triangles

Triangles in which all the three angles are identical. It is only necessary to determine that two sets of angles are identical in order to conclude that two triangles are similar; the third set will be identical because all of the angles of a triangle always sum to 180º.

xv) Proportionality of Similar Triangles

In similar triangles, the sides of the triangles are in some proportion to one another. For example, a triangle with lengths 3, 4, and 5 has the same angle measures as a triangle with lengths 6, 8, and 10. The two triangles are similar, and all of the sides of the larger triangle are twice the size of the corresponding legs on the smaller triangle.

xvi) 45-45-90 Triangle

A triangle that has angle measures of 45º, 45º, and 90º. The three sides in these triangle always fit into a specific proportion. If the side opposite either 45º angle is labeled *x*, then the side opposite the other 45º angle is also *x*, and the side opposite the 90º angle is $x\sqrt{2}$.

xvii) 30-60-90 Triangle

A triangle that has angle measures of 30º, 60º, and 90º. The three sides in these triangles always fit into a specific proportion. If the side opposite the 30º angle is labeled *x*, then the side opposite the 60º is $x\sqrt{3}$, and the side opposite the 90º angle is $2x$.

xviii) Parallelogram

A four-sided closed shape composed of straight lines in which the opposite sides are equal and the opposite angles are equal.

xix) Area of Parallelogram

Area = base * height, where the base refers to any side of the parallelogram and the height refers to the length of a line drawn from one of the opposite vertices to create a 90° angle with that base.

xx) Rectangle

A four-sided closed shape in which all of the angles equal 90° and in which the opposite sides are equal. Rectangles are also parallelograms but all parallelograms are not rectangles.

xxi) Area of Rectangle

Area = length * width, where length and width refer to the lengths of two adjacent sides of the rectangle.

xxii) Rhombus

A four-sided closed shape in which all of the sides are equal and in which the opposite angles are also equal. Rhombi are also parallelograms.

xxiii) Area of a Rhombus

Area = (Diagonal$_1$ * Diagonal$_2$) / 2, where the diagonals refer to the lengths of the lines drawn between opposite vertices in the rhombus.

xxiv) Square

A four-sided closed shape in which all of the angles equal 90° and all of the sides are equal. Squares also qualify as rectangles, rhombi, and parallelograms.

xxv) Diagonal of a Square

The diagonal of any square can be found by multiplying the length of one side by the square root of 2.

xxvi) Trapezoid

A four-sided closed shape in which one pair of opposite sides is parallel, but the other pair is not parallel.

xxvii) Area of a Trapezoid

Area = {(Base$_1$ + Base$_2$) * Height} / 2, where Base$_1$ and Base$_2$ refer to the two parallel sides, and the height refers to the length of a perpendicular line drawn between the two parallel bases.

xxviii) Rectangular Solid

A three-dimensional shape consisting of six faces, at least two of which are rectangles (the other four may be rectangles or squares, depending upon the shape's dimensions).

xxix) Main Diagonal of a Rectangular Solid

The main diagonal of a rectangular solid is the one that cuts through the center of the solid; the diagonal of a face of the rectangular solid is not the main diagonal.

The main diagonal of a rectangular solid can be found by using the "Deluxe" Pythagorean Theorem = $x^2 + y^2 + z^2 = d^2$, where x, y, and z are the length, width, and height of the rectangular solid, and d is the main diagonal.

xxx) Surface Area of a Rectangular Solid

Surface area = the sum of the areas of all six faces

xxxi) Volume of a Rectangular Solid

Volume = length * width * height, where length, width, and height refer to the three dimensions of the rectangular solid.

xxxii) Cube

A three-dimensional shape consisting of six identical faces, all of which are squares.

xxxiii) Surface Area of a Cube

Surface area = the area of any one face multiplied by 6.

xxxiv) Volume of a Cube

Volume = s^3, where *s* refers to the length of any one side of the cube.

xxxv) Main Diagonal of a Cube

The main diagonal of a cube is the one that cuts through the center of the cube; the diagonal of a face of a cube is not the main diagonal. The main diagonal of any cube can be found my multiplying the length of one side by the square root of 3.

Circles & Cylinders

i) Circle & Semi-Circle

A circle is a set of points in a plane that are equidistant from a fixed center point. Half of a circle = a semi-circle. A semicircle contains 180º, exactly half of the 360º in a circle.

ii) Chord

A line segment that connects any two distinct points on a circle's circumference.

iii) Radius

A line segment that connects the center of a circle with any point on that circle's circumference.

iv) Diameter

A line segment that passes through the center of a circle and whose endpoints lie on the circle.

v) Circumference of a Circle

The measure of the perimeter of a circle. The circumference of a circle can be found with this formula: $C = 2\pi r$, where C is the circumference, r is the radius, and π is a constant that equals approximately 3.14.

vi) Arc

A portion of a circle's circumference (above), delineated by any two points on the circle's circumference.

vii) Length of an Arc

The proportion of the central angle to 360 is the same as the proportion of the arc to the circle's circumference. For example, if the central angle is 60º, the proportion is 60/360 = 1/6. The arc, then, is 1/6 the length of the total circumference.

viii) Area of a Circle

The area inside the circle. The area of a circle can be found with this formula: Area = πr^2, where r is the radius of the circle and π is a constant that equals approximately 3.14.

ix) Inscribed Triangle

A triangle drawn inside another shape, such that the vertices of the triangle coincide with points on the edge of the other shape. For a circle, the vertices of the triangle are also points on the circle's circumference.

x) Right Triangle Inscribed in Circle

A right triangle inscribed in a circle must have its hypotenuse as the diameter of the circle. The reverse is also true: if the diameter of the circle is also the triangle's hypotenuse, then that triangle is a right triangle.

xi) Cylinder

Composed of two circles on either end of a rolled up rectangle.

xii) Surface Area of a Cylinder

Sum of the areas of the three shapes. The two circles have area πr^2, and the rectangle has area lw. The length of the rectangle is equal to the circumference of the circle ($2\pi r$) and the width is equal to the height of the cylinder (h). Thus, the total surface area of the cylinder = $2(\pi r^2) + 2\pi rh$.

xiii) Volume of a Cylinder

$V = \pi r^2 h$, where V is the volume, r is the radius of the cylinder, h is the height of the cylinder, and π is a constant that equals approximately 3.14.

Lines & Angles

i) Straight Line, or Line

A line measuring 180º. Extends indefinitely in both directions.

ii) Line Segment

A line segment has a finite length.

iii) Parallel Lines

Lines that lie in a plane and never intersect.

iv) Perpendicular Lines

Lines that intersect at a 90 angleº.

v) Intersecting Lines

Two lines that intersect, or cross, at a point.

vi) Interior Angles of Intersecting Lines

The interior angles of two intersecting lines form a circle (360º)

vii) Vertical Angles

The angles found opposite each other when two lines intersect; vertical angles are equal in measure.

viii) Exterior Angle

An angle created by extending the line of a polygon beyond the vertex of that polygon.

ix) Exterior Angle of a Triangle

The exterior angle of a triangle is equal to the sum of the two non-adjacent (or opposite) interior angles.

x) Transversal

A line that crosses, or cuts, two parallel lines; the transversal creates four angles where it crosses each parallel line, for a total of eight angles. All of the acute (or smaller) angles are equal and all of the obtuse (or larger) angles are equal. The sum of any acute angle and any obtuse angle is equal to 180º.

Coordinate Geometry

i) Coordinate Plane

Consists of a horizontal axis (typically labeled "*x*") and a vertical axis (typically labeled "*y*"), intersecting at the number zero on both axes.

ii) Coordinate Pair, or Ordered Pair

The values of a point on a number line. The first number in the pair is the *x*-coordinate, which corresponds to the horizontal location of the point as measured by the *x*-axis. The second number in the pair is the *y*-coordinate, which corresponds to the vertical location of the point as measured by the *y*-axis.

iii) Origin of a Coordinate Plane

The coordinate pair (0,0) represents the origin of a coordinate plane

iv) Slope of a Line

The slope is defined as "rise over run," or the distance the line runs vertically divided by the distance the line runs horizontally.

Slope = y_2-y_1 / x_2-x_1

The slope of any given line is constant over the length of that line.

v) Types of Slopes

A line can have one of four types of slope: positive, negative, zero, or undefined. When viewing a line from left to right, the slope is positive if the line rises and negative if the line falls. If the line is perfectly horizontal, the slope is zero. If the line is perfectly vertical, the slope is undefined.

vi) Intercept of a Line

A point where a line intersects a coordinate axis. If the line intersects the x-axis, the point is called the x-intercept, and the coordinate pair will take the form $(x,0)$. If the line intersects the y-axis, the point is called the y-intercept, and the coordinate pair will take the form $(0,y)$.

vii) Slope-Intercept Equation of a Line

The slope-intercept equation is $y = mx+b$, where x and y represent the x and y coordinates of a particular point, m represents the slope of the line, and b represents the y-intercept.

viii) Linear Equation

An equation that represents a straight line. An equation for a vertical line will take the form x = some constant number. An equation for a horizontal line will take the form y = some constant number.

Equations for all other lines will include both an x term and a y term, and these two terms will be connected by either an addition symbol or a subtraction symbol. In all cases, a linear equation will never use terms such as x^2, $\sqrt{y}$, or xy.

ix) Quadrants of a Coordinate Plane

The coordinate plane is divided into four quadrants. Quadrants are numbered beginning in the top right quadrant and moving counter-clockwise.

Quadrant I contains points with a positive x-coordinate and a positive y-coordinate. Quadrant II contains points with a negative x-coordiante and a positive y-coordinate. Quadrant III contains points with a negative x-coordinate and a negative y-coordinate. Quadrant IV contains points with a positive x-coordinate and a negative y-coordinate.

x) Bisector

A line or line segment that cuts another line segment exactly in half.

xi) Perpendicular Bisector

A line or line segment that cuts a line segment exactly in half and forms a 90º angle with that line.

xii) Slopes of Perpendicular Lines

Two perpendicular lines have negative reciprocal slopes. For example, given two perpendicular lines and a slope of 2/3 for one of the lines, to find the slope of the second line, first take the reciprocal of the given slope (so 2/3 becomes 3/2) and then reverse the sign. In this case, the given slope was positive, so the sign becomes negative. The negative reciprocal slope of 2/3 is -3/2.

xiii) Parabola

The graph of a quadratic function is in the shape of a parabola. A quadratic function is an equation containing exponents or roots. For example, $y = x^2$ is a quadratic function.

5) Statistics

i) Average (arithmetic mean)

$A = S / n$, where S is the sum of all of the terms in the set, n is the number of terms in the set, and A is the average.

ii) Weighted average

In a weighted average, some data points contribute more than others to the overall average. This is in contrast to a regular average, in which each data point contributes equally to the overall average.

A weighted average can be expressed with the formula $A = [(D_1)(W_1) + (D_2)(W_2) + \ldots + (D_n)(W_n)]$ / sum of weights, where each D represents a distinct data point, each W represents the weighting assigned to that data point, and A is the weighted average.

iii) Median

Literally, the "middle" value in a set of numbers written in increasing (or decreasing) order. In a set with an odd number of terms, the median is the middle number. In the set 1, 3, 4, 6, 9, the median is 4. In a set with an even number of terms, the median is the average of the two middle numbers. In the set, 1, 3, 4, 6, the median is (3+4)/2 = 3.5.

iv) Mode

The mode of a distribution is the value of the term that occurs the most often. It is not uncommon for a distribution with a discrete random variable to have more than one mode, especially if there are not many terms. This happens when two or more terms occur with equal frequency, and more often than any of the others.

v) Standard Deviation (SD)

A measurement used to describe the how far apart numbers in a set are. This is also called the "spread" or the "variation" of the set. Technically, SD is a measure of how far from that set's average the data points typically fall. SD can be either positive or zero.

- A small SD indicates that the terms of the set are clustered closely around the average value of that set.
- A large SD indicates that the terms of the set are widely spread, with some terms very far from the average value of that set.
- An SD of zero indicates that all of the terms of that set are exactly equal to that set's average.

vi) Ratio

Generally expresses a "part to part" relationship, though a ratio can be hidden within a "part to whole" relationship.

Part to part: Given that there are 3 oranges for every 2 apples in a basket, the ratio of oranges to apples is 3 parts to 2 parts, or 3:2, or (3/2). The "whole" is 3+2 = 5; there are 5 pieces of fruit in each "group" of 3 oranges and 2 apples.

"Hidden" part to part relationship might be presented as: Given only apples and oranges in a basket, 3 out of every 5 pieces of fruit are oranges. Because the fruit must be either apples or oranges, there must be 3 oranges for every 2 apples in order to arrive at the "whole" 5 pieces of fruit. Thus, the ratio of oranges to apples is 3 parts to 2 parts, or 3:2 or (3/2).

vii) Proportion

Generally, a ratio presented in the form of an equation. Given a ratio of 3 oranges to 2 apples, and an actual total of 15 oranges, a proportion would read: (3 oranges / 2 apples) = (15 oranges / ? apples).

Note: The ratio and the actual numbers are in proportion; the actual numbers always simplify to the ratio.

viii) Fundamental Counting Principle

When making a number of separate decisions, multiply the number of ways to make each individual decision in order to find the number of ways to make all of the decisions. Given a choice of 3 candidates for President and 2 candidates for Treasurer, there are 3*2 = 6 possible combinations for the President – Treasurer team.

ix) Factorial (!)

The symbol for factorial is the exclamation point (!). $n!$ is the product of all integers less than or equal to n. 4! = 4 * 3 * 2 * 1 = 24. In practice, the number 1 can be ignored (because 1 multiplied by anything does not change the product).

x) Combination

A subset of items chosen from a larger pool in which the order of items does not matter. When choosing 5 people from a pool of 10 to be on a basketball team, it doesn't matter if Juliette is chosen first and Susie is chosen second or vice versa; if they are both chosen, in any order, then both are part of that 5-person team.

The formula is $n! / [(n - r)! * r!]$, where n is the total number of items in the pool and r is the number of items to be chosen. In the above example, $n = 10$ and $r = 5$.

xi) Permutation

A subset of items chosen from a larger pool in which the order of items does matter. Ten people are running a race and three ribbons are to be awarded to the first (blue), second (red), and third (yellow) place finishers. If Juliette finishes first and Susie finishes second, this is a different scenario than Juliette finishing second and Susie finishing first; they are going to receive different ribbons depending upon how they finish.

The formula is $n! / (n - r)!$, where n is the total number of items in the pool and r is the number of items to be chosen. In the above example, $n = 10$ and $r = 3$.

xii) Probability

Probability is the likelihood that a certain event will occur. The formula is: probability = (number of desired or successful outcomes) / (total number of possible outcomes).

The probability that some event definitely will occur is 1, or 100%. The probability that some event definitely will not occur is 0, or 0%. Further, the collection of all possible outcomes of a particular scenario add up to a probability of 1, or 100%.

xiii) "And" in probability

To determine the probability that event *A* and event *B* will both occur, find the individual probabilities of each event and multiply them together. "and" = "multiply".

xiv) "Or" in probability

To determine the probability that event *A* or event *B* will occur (and assuming that event *A* and event *B* cannot both happen together), find the individual probabilities of each event and add them together. "or" = "add" .

Note*:* If event *A* and event *B* could both occur together, then to calculate the probability that either event *A* or event *B* occurs, find the individual probabilities of each event, add them together, and then subtract the probability that the two events will happen together.

5) Decimals & Fractions

i) Decimal

Numbers that fall in between integers; expresses a part-to-whole relationship in terms of place value.

Example: 1.2 is a decimal. The integers 1 and 2 are not decimals. An integer written as 1.0, however, is considered a decimal.

ii) Digit

There are ten digits that make up all numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The three-digit number 412 consists of the digits 4, 1, and 2.

iii) Place Value

Every digit in a given number has a particular place value. The place value depends upon the digits location relative to the decimal point.

6	7	8	9	1	0	2	3	.	8	3	4
TEN MILLIONS	ONE MILLION	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES	DECIMAL	TENTHS	HUNDRETHS	THOUSANDTHS

iv) Place Value and Powers of 10

Place values decrease from left to right by powers of 10.
Table:

Words	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
Numbers	1000	100	10	1	0.1	0.01	0.001
Powers of 10	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

v) Rounding

Simplifying a number to a certain place value. Drop the extra decimal places, and if the first dropped digit is 5 or greater, round up the last digit that you keep. If the first dropped digit is 4 or smaller, round down (keep the same) the last digit that you keep.

Example: 9.1278 rounded to the nearest tenth = 9.1, since the dropped 2 is less than 5.

9.1278 rounded to the nearest hundredth = 9.13, since the dropped 7 is greater than (or equal to) 5.

9.1278 rounded to the nearest thousandth = 9.128, since the dropped 8 is greater than (or equal to) 5.

vi) Adding or Subtracting Decimals

Write the problem vertically and line up the decimal points. Add any necessary zeroes to the right side of any numbers in order to make the numbers the same length.

Example:

$$\begin{array}{r} 71.2 \\ \underline{+184.99} \\ 256.19 \end{array}$$

vii) Multiplying Decimals

Drop the decimal points and multiply normally (as you would multiply whole numbers). At the end, count the total number of digits to the right of the decimal in the original numbers. Insert the same number of decimal places into the answer.

Example:

0.7 * 3 = ?

First, multiply normally: 7*3 = 21

Then count the decimals represented in the original numbers; in this case, we have one decimal among the original numbers.

Insert one decimal into the answer, 21, to come up with 2.1

viii) Dividing Decimals (Dividend)

If there is a decimal in the dividend (the inner number), bring the decimal point up to the answer and then divide normally.

Example:

```
     3.09
4 | 12.36
     12
       03
        0
        36
```

ix) Re-writing Decimals Using Powers of 10

Decimals can be re-written in terms of powers of 10 or vice versa.

Example:

$0.006 = 6 * 10^{-3}$

x) Terminating Decimals

Decimals that terminate, or end, at some point or decimals that do not go on forever. Eg. 13.2 is a terminating decimal whereas 13.33 does not terminate but goes on and on. π also does not terminate.

xi) Fraction

A way to express numbers that fall in between integers (though integers can also be expressed in fractional form). A fraction expresses a part-to-whole relationship in terms of a numerator (the part) and a denominator (the whole).

Example: (7/2) is equivalent to the decimal 3.5.

xii) Numerator & Denominator

The top part of a fraction is called the numerator and the bottom part the denominator. In the fraction (7/2), 7 is the numerator and 2 is the denominator.

xiii) Proper Fraction

Fractions that have a value between 0 and 1. The numerator is always smaller than the denominator. Eg. (1/2) is a proper fraction whereas (3/2) is not a proper fraction.

xiv) Improper Fraction

Fractions that are greater than 1. These can also be written as a mixed number. (7/2) is an improper fraction. This can also be written as a mixed number: 3 (1/2).

xv) Mixed number

An integer combined with a proper fraction. These can also be written as an improper fraction. 3 (1/2) is a mixed number. This can also be written as an improper fraction: (7/2)

xvi) Complex fraction

A fraction in which there is a sum or difference in the numerator or denominator.

Example:

- (3+6)/10
- 10/(3+6)

xvii) Simplifying Fractions

Reducing numerators and denominators to the smallest form. Dividing the numerator and denominator by the same number does not change the value of the fraction.

Example:

Given (21/6), we can simplify by dividing both the numerator and the denominator by 3. The simplified fraction is (7/2).

xviii) Reciprocal

The product of a number and its reciprocal is always 1. To get the reciprocal of an integer, put that integer on the denominator of a fraction with numerator 1. The reciprocal of 3 is (1/3).

Note:

To get the reciprocal of a fraction, switch the numerator and the denominator. The reciprocal of (2/3) is (3/2).

xix) Dividing Fractions

Change the divisor into its reciprocal and then multiply.

Example:

Given (3/5) / 2, take the reciprocal of 2. The reciprocal is (1/2). Now multiply: (3/5) * (1/2) = (3/10).

xx) Common Denominator

When adding or subtracting fractions, we first must find a common denominator, generally the smallest common multiple of both numbers.

Example:

Given (3/5) + (1/2), the two denominators are 5 and 2. The smallest multiple that works for both numbers is 10. The common denominator, therefore, is 10.

xxi) Adding or subtracting fractions

Adding or subtracting fractions: always simplify within a given numerator and denominator, but do NOT simplify across fractions. Instead, find a common denominator first and only then combine the two (or more) numerators.

Example:

Given (3/5) + (2/4), we can simplify (2/4) to (1/2). The simplified problem is (3/5) + (1/2). The common denominator is 10. Multiply the first fraction by 2 and the second fraction by 5 to get (6/10) + (5/10). Next, add the numerators and keep the same denominator to get (11/10).

5) Miscellaneous Concepts

i) Percent

Literally, "per one hundred". Percent expresses a special part-to-whole relationship between a number (the part) and one hundred (the whole). A special type of fraction or decimal that involves the number 100.

ii) Finding a percentage of a number

What is 25% of 40?

Translation: n = (25/100)*40.

30% of what is 50?

(30/100) * n = 50?

iii) Percent increase or decrease, or percent change

The percentage of the starting point that some number increased or decreased.

Formula: $\frac{\text{Change}}{\text{Original}} = \frac{\text{Percent}}{100}$

Example:

Given a starting point of 10 and an ending point of 14, the change is 4 and the original is 10, so the percent increase is (4/10) = (2/5) = 40%.

iv) Simple Interest

Formula: Simple Interest = Principal Amount * Rate * Time /100

Example:

Given $1,000 invested for 6 years at an annual rate of 4%, $1,000 is the principal amount, 4 is the rate per year (annual), and 5 years is the time (expressed in the same units, years, as the time used in the rate).

Thus SI = 1000*4*5/100 = $200 per annum

vi) Compound Interest

Formula: $P * (1 + (r/n))^{nt}$

Example:

Given $1,000 invested for 2 years, compounded semi-annually, at an annual, or yearly, rate of 4%, $1,000 is the principal amount, 0.04 is the rate, *n* is 2 (because it is compounded twice in a year), and *t* is 2 (because the money is invested for 2 years). The above problem is expressed as:

$1,000 * (1 + (0.04 / 2))^(2 * 2) = $1,082.43

vii) Rate formula

Rate * Time = Distance, where Rate is measured in units of distance per unit of time (for example, miles per hour).

viii) Work Formula

Rate * Time = Work, where Rate is measured in units of output per unit of time (for example, 5 cars produced per hour).